Received: *October 09, 2000*, Accepted: *November 30, 2000*

$\mathbb{Z}_2$ monopoles in $D = 2+1$ SU(2) lattice gauge theory

Alistair Hart[*]

Department of Physics and Astronomy, University of Edinburgh
Edinburgh EH9 3JZ, Scotland, UK
E-mail: hart@ph.ed.ac.uk

Biagio Lucini and Mike Teper

Theoretical Physics, University of Oxford, 1 Keble Road
Oxford OX1 3NP, England, UK
E-mail: lucini@thphys.ox.ac.uk, m.teper@physics.oxford.ac.uk

Zsolt Schram

Department of Theoretical Physics, University of Debrecen
H-4010 Debrecen P.O. Box 5, Hungary, and
Center for Computational Physics, University of Tsukuba
Tsukuba, Ibaraki 305-8577, Japan
E-mail: schram@atomki.hu

Abstract: We calculate the euclidean action of a pair of $\mathbb{Z}_2$ monopoles (instantons), as a function of their spatial separation, in $D = 2+1$ SU(2) lattice gauge theory. We do so both above and below the deconfining transition at $T = T_c$. At high T, and at large separation, we find that the monopole "interaction" grows linearly with distance: the flux between the monopoles forms a flux tube (exactly like a finite portion of a $\mathbb{Z}_2$ domain wall) so that the monopoles are linearly confined. At short distances the interaction is well described by a Coulomb interaction with, at most, a very small screening mass, possibly equal to the Debye electric screening mass. At low T the interaction can be described by a simple screened Coulomb (i.e. Yukawa) interaction with a screening mass that can be interpreted as the mass of a "constituent gluon". None of this is unexpected, but it helps to resolve some apparent controversies in the recent literature.

Keywords: Solitons Monopoles and Instantons, Confinement, Duality in Gauge Field Theories, Lattice Gauge Field Theories.

[*]Current address: DAMTP, University of Cambridge, Cambridge CB3 0WA, UK

JHEP11(2000)043

Contents

1. Introduction

In a recent paper [1] the free energy of a separated pair of $\mathbb{Z}_2$ monopoles has been calculated in $D = 3+1$ SU(2) lattice gauge theory. This problem is difficult because it involves the numerical evaluation of the ratio of two different partition functions, so that the calculations in [1] are only accurate at small monopole separation. The conclusion is that the interaction potential has a simple Coulomb/Yukawa form both at high and at low T; and that this remains so, at high T, even if one considers monopoles whose world-lines are spacelike rather than timelike. While the other results are consistent with expectations, this last is not. What one expects is that the $\mathbb{Z}_2$ flux between the monopoles will form a "flux tube" so that the interaction energy grows linearly at large separation. Motivated by this unexpected result we have performed similar calculations using a different method that is less direct but much more accurate. This method has been used recently [2, 3] in closely related investigations of vortices [4] and, some time ago, of high-T $\mathbb{Z}_2$ domain walls [5]. Our calculations are carried out in $2+1$ euclidean dimensions where, apart from the fact that the monopoles are instanton-like rather than soliton-like, not only can the same questions be posed as in $3+1$ euclidean dimensions but they can be answered much more accurately. In addition the close relationship between $\mathbb{Z}_2$ monopoles, vortices and domain walls is particularly transparent in $2+1$ dimensions. (Although it holds in $D = 3+1$ equally well.) As we shall emphasise, after a suitable relabelling of the space-time axes, one can see [2, 5] that the high-T $\mathbb{Z}_2$ domain wall is a periodic vortex line (as created by a periodic 't Hooft disorder loop [4]) since this phase is a Higgs phase for vortices. And in this phase the $\mathbb{Z}_2$ flux between a pair of well-separated monopoles passes through a flux tube that is nothing but a portion of such a domain wall that ends on these monopoles; so that such $\mathbb{Z}_2$ monopoles are "linearly confined". We shall indeed confirm that the interaction is linear at high T, as expected. The contrary conclusion in [1] can presumably be attributed to the lack of accurate calculations at larger inter-monopole separation. We have not pursued the same calculations in $D = 3+1$ since, as our $D = 2+1$ calculations were completed, a paper appeared [6] addressing similar questions in $D = 3+1$ with conclusions similar to ours (where they overlap).

In the next section we describe $\mathbb{Z}_2$ monopoles and explain how we expect them to behave. We then describe our Monte Carlo calculations and finish with some conclusions. Since most of the techniques and physical quantities involved have been described in detail in earlier papers [1, 2, 5], we shall aim to be succinct rather than pedagogical in this brief report.

2. $\mathbb{Z}_2$ monopoles-expectations

In $2+1$ euclidean dimensions, the monopoles are localised in space and time and so can equally well be thought of as instantons. What we shall do is calculate the extra euclidean action, ΔS, associated with the presence of a pair of such monopoles. If we were in $D = 3+1$ this quantity would be proportional to the derivative of the free energy of the monopole pair, and its variation with the monopole distance would tell us how the monopole interaction energy varies with distance. In $D = 2+1$ we shall, by analogy, sometimes refer to the action difference as the monopole "interaction energy".

To calculate the interaction between $\mathbb{Z}_2$ monopoles at sites x and x' (on the dual lattice) one introduces a $\mathbb{Z}_2$ Dirac string that starts at x and ends at x' [7]. Such a string flips the sign of any plaquette that it threads. (This is so if, as here and in the related literature, the SU(2) group elements on the links are represented by fundamental 2×2 matrices rather than by adjoint 3×3 matrices.) One can introduce such a Dirac string by changing $\beta \to -\beta$ on the plaquettes threaded by the string. We shall refer to this as a set of "twisted" plaquettes because, in the case where one obtains x' by translating x by a single lattice period, this is equivalent to the imposition of twisted boundary conditions. By redefining suitable subsets of link matrices by $U' = -U$ [1, 2, 5], it is easy to see that the actual location of the string, apart from its end points, is invisible; as we would wish for a Dirac string.

2.1 $T > T_c$

Let us first consider the extreme case where the line of twisted plaquettes extends right across the lattice; say in the x-direction with the twisted plaquettes in the (y,t) plane. This produces a $\mathbb{Z}_2$ Dirac string that closes upon itself through the boundary;

so it is not attached to any monopoles. Such a twist is, as remarked earlier, equivalent to imposing twisted boundary conditions. For $T > T_c$ it imposes the presence of a $\mathbb{Z}_2$ domain wall separating the two high-T $\mathbb{Z}_2$ vacua [5]. As pointed out in [2, 5], this domain wall is in fact a periodic spacelike vortex tube that has been squashed by the narrow extent in the euclidean time direction. Its existence demonstrates that such spacelike $\mathbb{Z}_2$ flux is, at high T, confined to a flux tube. So we expect that for $T > T_c$ the interaction energy between two spacelike separated $\mathbb{Z}_2$ monopoles will grow linearly with their separation, r, once this separation is larger than the width of the tube

$$\Delta S \simeq \bar{\sigma} r \qquad r \text{ large}\,. \tag{2.1}$$

(Note that all quantities will be expressed in lattice units from now on.) At small separations the flux of one monopole spreads out in the three euclidean space-time directions and then returns to the other monopole with no opportunity to form a flux tube. Thus the interaction should be Coulombic $\propto 1/r$. As r is increased we might expect a Yukawa-type correction

$$\Delta S \propto \frac{e^{-\mu r}}{r} \qquad r \text{ small}\,, \tag{2.2}$$

with the scale μ being plausibly related to the Debye electric screening mass [5]. The latter governs the width of the vortex tube/domain wall which, for large r, carries the return flux between the monopoles over most of the distance between them.

Note that if there were no linear term then the asymmetric geometry of our system would mean that, as we increased r to $r > 1/T$, any $1/r$ Coulomb term would turn into a logarithm $\propto \log(rT)$. This is also the case if we consider spatially separated $\mathbb{Z}_2$ monopoles at high T in $3+1$ dimensions. We shall ignore this for the purposes of our fits because, in practice, the Coulomb or Yukawa contribution becomes very small for $r > 1/T$.

At very high T we expect the tension of the $\mathbb{Z}_2$ domain wall to be calculable in perturbation theory and, as shown in [5], the perturbative expression is accurate down to remarkably small values of T/g^2. (Recall that in $D = 2+1$ g^2 has dimensions of mass.) Thus the fact that our spacelike separated monopoles are joined by a section of such a domain wall, allows us to estimate the value of $\bar{\sigma}$ in eq. (2.1):

$$\bar{\sigma} = 2\alpha \left(\frac{T}{g^2}\right)^{3/2} \frac{1}{\beta^2}\,, \tag{2.3}$$

where β is the usual (inverse) lattice coupling and

$$\alpha = \alpha_{\text{pert}} \left(1 + 0.25\frac{g^2}{T}\right) \tag{2.4}$$

provides an accurate representation of the Monte Carlo calculated domain wall tension for $g^2/T < 1.2$. The leading order perturbative value, α_{pert}, can be calculated as a function of the temporal lattice discretisation and the resulting values are listed in [5, table 8].

In addition to the above quantitative expectation for the interaction between spatially separated monopoles at high T, one might equally well consider the interaction of two monopoles with a timelike separation. However, because such a separation is limited to very small values, $r \leq 1/2T$, there is nothing very interesting to be learned and we shall not consider that case any further here.

2.2 $T < T_c$

Consider now the low-T confining phase. In this case if there are vortex tubes, they condense into the vacuum and thus the interaction between monopoles becomes independent of r at large r. At small r we again expect a screened Coulomb interaction $\propto \exp(-\mu r)/r$, or a sum of such terms. But now there is no T to set the scale, so (the lightest) μ will presumably be related to the mass gap, which is provided by the lightest 0^{++} glueball in this theory [8].

The above discussion translates directly to $D = 3 + 1$. Now the $\mathbb{Z}_2$ monopoles will have world lines and the Dirac string will sweep out a sheet, which requires a sheet of twisted plaquettes. If we are at high T and the monopole world lines and separations are spacelike, then the return flux once again goes through flux tubes, with the scale set by the electric Debye mass. So at large separations the interaction energy should grow linearly and at small distances it will be screened Coulombic. By contrast, at low T the vortex tubes condense in the vacuum and there is only a screened Coulombic interaction. While the calculations of [1] agree with the above expectations at low T they claim that a simple Coulomb interaction, with no linear piece, works at high T as well. Our method of using the action difference is, however, much easier and more accurate, especially at larger r, than the method employed in [1] and we now turn to calculations designed to resolve this puzzle.

3. $\mathbb{Z}_2$ monopoles-calculations

Our Monte Carlo calculations use the usual plaquette action parameterised by the (inverse) coupling β where $\beta \to 4/ag^2$ as $a \to 0$. The low-T physics of this theory is discussed in [8], the properties at high T in [5], and the deconfining transition in [9].

We perform calculations at two values of the coupling: $\beta = 9$ and $\beta = 13.5$. In each case we perform one calculation at $T > T_c$ and one at $T < T_c$. At $\beta = 9$ we perform calculations on $24^2 4$ and $24^2 8$ lattices and at $\beta = 13.5$ on $36^2 6$ and $36^2 12$ lattices. On a $L^2 L_t$ lattice of spatial extent $L \gg L_t$ we have $aT = 1/L_t$ so the above choices correspond to higher and lower T, respectively. To be more quantitative we recall that the critical value of L_t that corresponds to $T = T_c$ is given, as a function of β, by [2]

$$L_t^c(\beta) \simeq \frac{1}{1.55}(\beta - 0.37)\,. \tag{3.1}$$

We see that, at both values of β, the lower value of T corresponds to $T \simeq 0.7T_c$ while the higher value corresponds to $T \simeq 1.4T_c$.

For each of these lattices we calculate the average action with a line of twisted plaquettes of length r which introduces two $\mathbb{Z}_2$ monopoles a distance r apart as discussed earlier. For each value of r we perform a Monte Carlo calculation, typically with 10^6 sweeps, of the average total action, $\langle S(r)\rangle$. The action here is $S = \sum_p (1 - \frac{c_p}{2} TrU_p)$ where U_p is the product of the four SU(2) matrices around the plaquette p, and c_p is -1 for the twisted plaquettes and $+1$ otherwise. We plot in tables 1 and 2 our results for the action. We turn now to a discussion of these results.

3.1 $T > T_c$

In figure 1 we plot

$$\Delta S = \langle S(r)\rangle - \langle S(0)\rangle \tag{3.2}$$

for the $\beta = 13.5$ high-T calculation. We observe an unambiguous confirmation of the expected linear rise with r at large r. Note that the point at $r = 36$ has not been misplaced. Its value is given by a periodic flux tube with no monopoles attached; thus it is missing the large (divergent as $a \to 0$) Coulombic self-energy of the monopoles, that otherwise contributes to $S(r)$ (except when r is close to the lattice period).

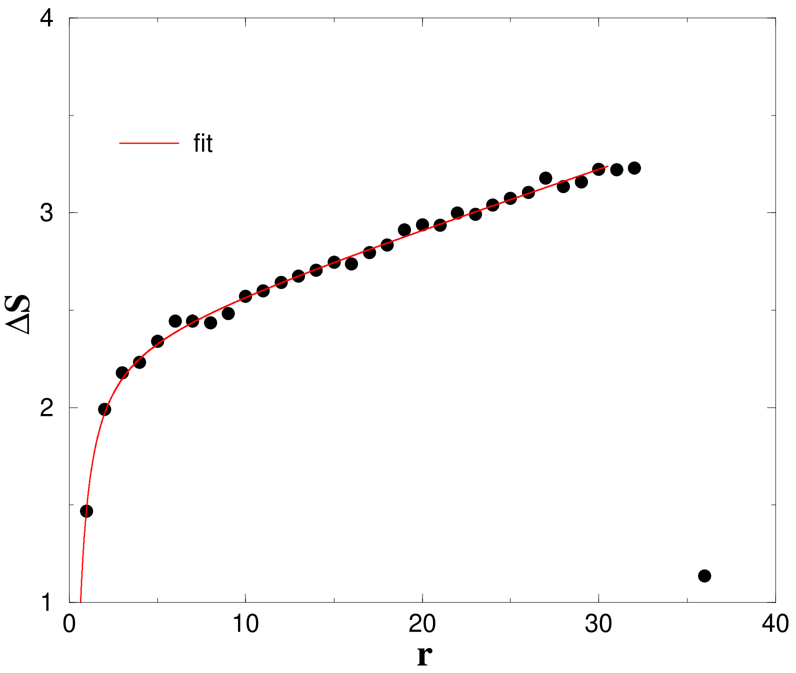

Figure 1: The action of two $\mathbb{Z}_2$ monopoles a spacelike distance r apart on a $36^2 6$ lattice at $\beta = 13.5$ for which $T > T_c$.

S ; $\beta = 9$		
r	$24^2 4$	$24^2 8$
0	789.974(28)	1583.412(34)
1	791.492(26)	1584.839(34)
2	792.039(26)	1585.299(34)
3	792.205(25)	1585.423(34)
4	792.351(25)	1585.536(29)
5	792.443(29)	1585.471(39)
6	792.513(28)	1585.542(36)
7	792.640(28)	1585.560(34)
8	792.718(28)	1585.488(36)
9	792.825(23)	1585.521(36)
10	792.879(25)	1585.578(41)
11	792.948(26)	1585.628(32)
12	793.059(27)	1585.507(37)
13	793.191(24)	
14	793.248(26)	
15	793.302(25)	
16	793.411(25)	
17	793.429(25)	
18	793.560(23)	
19	793.579(25)	
20	793.657(24)	
21	793.701(18)	
22	793.639(24)	
23	793.223(24)	
24	791.737(25)	

Table 1: The total lattice action, S, on $24^2 4$ and $24^2 8$ lattices at $\beta = 9$, as a function of the separation, r, between the two $\mathbb{Z}_2$ monopoles.

S ; $\beta = 13.5$		
r	$36^2 6$	$36^2 12$
0	1760.398(29)	3522.752(39)
1	1761.865(32)	3524.321(49)
2	1762.390(28)	3524.750(42)
3	1762.577(30)	3524.983(44)
4	1762.630(27)	3525.077(41)
5	1762.739(31)	3525.050(44)
6	1762.841(30)	3525.081(45)
7	1762.843(33)	3525.140(41)
8	1762.832(31)	3525.095(45)
9	1762.882(24)	3525.004(38)
10	1762.969(30)	3525.006(40)
11	1762.998(27)	3525.118(47)
12	1763.041(29)	3525.038(49)
13	1763.072(30)	3525.049(45)
14	1763.102(28)	3525.152(41)
15	1763.144(31)	3525.074(39)
16	1763.134(26)	3525.088(41)
17	1763.195(31)	
18	1763.234(28)	
19	1763.311(30)	
20	1763.336(29)	
21	1763.335(29)	
22	1763.397(29)	
23	1763.390(28)	
24	1763.437(29)	
25	1763.472(27)	
26	1763.501(28)	
27	1763.576(29)	
28	1763.533(32)	
29	1763.557(30)	
30	1763.622(28)	
31	1763.619(30)	
32	1763.629(32)	
36	1761.537(28)	3522.863(45)

Table 2: The total lattice action, S, on $36^2 6$ and $36^2 12$ lattices at $\beta = 13.5$, as a function of the separation, r, between the two $\mathbb{Z}_2$ monopoles.

JHEP11(2000)043

We parameterise ΔS using a sum of linear and Yukawa terms

$$\Delta S = c_0 + c\frac{e^{-\mu r}}{r} + \bar{\sigma} r \,. \tag{3.3}$$

This conventional parameterisation is clearly approximate. We do not expect the linear piece to be there at all at small values of r. In addition, as remarked earlier, the Coulomb $1/r$ should mutate into a $\log r$ dependence at larger r. Moreover we expect a string/roughening correction to the linear term at large r, which can be calculated in perturbation theory at high T [5]. All of these corrections to eq. (3.3) are small and we shall assume, perhaps optimistically, that they are negligible for our purposes.

We have performed fits with eq. (3.3) to, for example, the range $r \in [1, 30]$. We exclude $r = 0$ from the fit for obvious reasons and we also exclude values $r > 30$ since as r approaches the lattice period, at some point it becomes energetically favourable for the "Coulombic" part of the monopole-monopole flux to join through the periodic boundary and for this to be matched by a periodic flux tube whose location is now decoupled from the location of the twist. We find that fits as in eq. (3.3) possess a good χ^2 — about 0.9 per degree of freedom for the best fit shown in figure 1. There are two immediate questions: is the monopole string tension as expected and what is the screening mass?

First, consider the monopole string tension. From the fit in eq. (3.3) we obtain a value

$$\bar{\sigma} = 0.0300 \pm 0.0009 \tag{3.4}$$

which is consistent with the value obtained from the $r = 36$ value: $\bar{\sigma} = 0.0315(12)$. We can compare this with our high-T theoretical expectation in eq. (2.3). Plugging in $\beta = 13.5$, $T/g^2 = \beta/4L_t = 13.5/24$, and the value of α from eq. (2.4) using the $L_t = 6$ value of α_{pert} as listed in [5, table 8] we obtain an expected value $\bar{\sigma} \simeq 0.035$. If instead of employing the heuristic correction term in eq. (2.4) we use only the one-loop value for α then we obtain $\bar{\sigma} \simeq 0.024$. Thus our fitted value of $\bar{\sigma}$ is in good agreement with our expectations; remarkably so given that the one-loop perturbative expression has been derived for much higher T than the values being considered here.

A similar analysis of the $\beta = 9$ calculation on a $24^2 4$ lattice yields best fits with a poorer, albeit not unacceptable, χ^2. We obtain a monopole string tension $\bar{\sigma} = 0.078(3)$ which is consistent with the string tension one finds with a periodic twist, $\bar{\sigma} = 0.0735(15)$. This can be compared with what one obtains from high-T perturbation theory, eqs. (2.3): $\bar{\sigma} \simeq 0.056$ without the heuristic $O(g^2/T)$ correction in eq. (2.4), and $\bar{\sigma} \simeq 0.081$ if we include it. Satisfactory agreement, in other words.

We turn now to the screening mass μ. Its fitted value is:

$$\mu = 0.000 \pm 0.044 \,, \qquad \beta = 13.5 \,. \tag{3.5}$$

That is to say, we are quite consistent with a zero mass and so a purely Coulombic (unscreened) short distance interaction between the monopoles. Certainly the screening mass is much less than the $T = 0$ mass gap. At high T there are additional natural mass scales, the simplest being $aT_c \simeq 0.12$, $aT = 1/6$, $\sqrt{g^2T} \simeq 0.22$. All of these are probably too large to be consistent with our value of μ. There are other more dynamical mass scales such as the (electric) Debye screening mass, m_D, and the spatial string tension σ_s. We have performed an explicit calculation of Polyakov loop correlations on the same $36^2 6$ lattice at $\beta = 13.5$ in order to determine these masses [5], and we find

$$m_D = 0.103 \pm 0.005 \,, \qquad \beta = 13.5 \tag{3.6}$$

and

$$\sqrt{\sigma_s} = 0.139 \pm 0.002 \,, \qquad \beta = 13.5 \,. \tag{3.7}$$

The value of $\sqrt{\sigma_s}$ is too large for μ but there is perhaps a possibility, at the 2σ level, that the Debye screening mass provides this scale.

A similar analysis of the $\beta = 9$ calculation yields

$$\mu = 0.2 \pm 0.2 \,, \qquad \beta = 9.0 \,. \tag{3.8}$$

This is again consistent with the (electric) Debye screening mass that we have calculated on the same lattice at the same β,

$$m_D = 0.16 \pm 0.01 \,, \qquad \beta = 9.0 \tag{3.9}$$

although, because of the larger errors, it is also consistent with most of the high-T scales referred to earlier (but not to the $T = 0$ mass gap).

The value of μ is hard to determine numerically because, as we see from figure 1, even at our relatively large value of β our resolution of the region in which the "Coulombic" interaction dominates is poor. We have tried fitting the values at small r with a purely Yukawa form, and that again produces a value consistent with zero. We have also modified the form in eq. (3.3) by multiplying the linear piece by a factor $1 - \exp(-\mu' r)$. This is quite natural; after all one would expect the linear term to gradually disappear once r becomes less than the flux tube width. Such fits give values of μ and μ' that are non zero and consistent with the value of m_D in eq. (3.6).

3.2 $T < T_c$

In figure 2 we plot the action, ΔS, of the two $\mathbb{Z}_2$ monopoles as a function of their separation, r, in the low-T confining phase at $\beta = 13.5$. In contrast to the situation in the deconfined high-T phase, we see that there is no linearly confining interaction between the monopoles. This is as expected. spacelike $\mathbb{Z}_2$ vortex tubes disorder timelike Wilson loops, and, in the confining phase, will condense in the vacuum, thus costing no extra action.

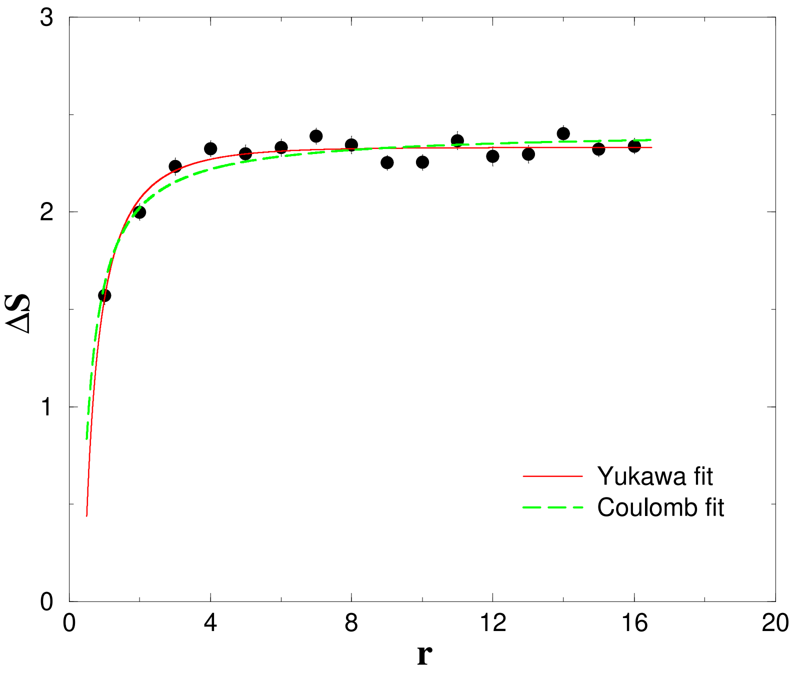

Figure 2: The action of two $\mathbb{Z}_2$ monopoles a spacelike distance r apart on a $36^2 12$ lattice at $\beta = 13.5$ for which $T < T_c$.

At small separations the monopoles feel a Coulomb interaction which should be screened at larger r in this phase. Thus we fit ΔS to the Yukawa form:

$$\Delta S = c_0 + c\frac{e^{-\mu r}}{r} \,. \tag{3.10}$$

We find that an unscreened Coulomb interaction is statistically unacceptable (see the fits shown in figure 2); there is clearly a non-zero screening mass

$$\mu = 0.4 \pm 0.1 \,, \qquad \beta = 13.5 \,. \tag{3.11}$$

Similar low-T calculations on the $24^2 8$ lattice at $\beta = 9$ also reveal a zero string tension and a non-zero screening mass:

$$\mu = 0.43 \pm 0.09 \,, \qquad \beta = 9.0 \,. \tag{3.12}$$

Since the above masses are in lattice units one would expect the $\beta = 13.5$ mass to be about two-thirds of the $\beta = 9$ value; which is quite possible within the rather large errors. We also observe that taken together the values in eqs. (3.11) and (3.12) are not really compatible with the $T = 0$ mass gap [8]: $m_G \simeq 0.495(10)$ at $\beta = 13.5$ and $m_G \simeq 0.76(1)$ at $\beta = 9$. This is reminiscent of what has been found [10] for the screening of the abelian monopole flux in the maximally abelian gauge in $D = 3+1$. The screening mass there was interpreted as the constituent gluon mass: half the average mass of the lightest 0^+ and 2^+ glueballs. We note that our screening masses, in eqs. (3.11) and (3.12), are consistent [8] with being interpreted in this way.

JHEP11(2000)043

4. Conclusions

We have shown that in the low-T confining phase the interaction of a pair of $\mathbb{Z}_2$ monopoles a distance r apart can be described by a simple Yukawa interaction $\propto \frac{1}{r}\exp(-\mu r)$ where the effective screening mass μ is consistent with what one might expect for the mass of a "constituent gluon" [10]. In the high-T deconfined phase the $\mathbb{Z}_2$ flux between widely separated monopoles is collimated in a flux tube and the interaction energy increases linearly with distance, i.e. the monopoles are "linearly confined". Such a spacelike flux tube, when closed upon itself through a spatial boundary, becomes a domain wall separating two high-T $\mathbb{Z}_2$ vacua. Its string tension can be calculated in perturbation theory at very high T, since the dimensionless coupling, g^2/T, becomes small there. The string tension we obtain is surprisingly close to the value expected from one-loop perturbation theory, given that in our "high-T" calculations g^2/T is in fact close to unity. At small r, where the flux tube has not yet formed, we find that the interaction is consistent with a Coulomb interaction possibly modified by very weak screening. This small screening mass is just about consistent with the Debye electric screening mass that we have also calculated. All these calculations are for values of the lattice spacing a that are sufficiently small, expressed either in units of g^2 or in units of T, that we can expect negligible corrections when taking the continuum limit.

All our calculations have been for monopoles in $2+1$ euclidean dimensions where monopoles are instantons rather than solitons. However we expect things to be very much the same in $3+1$ euclidean dimensions. At high T there are, again, perturbatively calculable "domain walls" between differing $\mathbb{Z}_2$ vacua, and these walls are just the regions swept out by a periodic $\mathbb{Z}_2$ flux tube as it moves through space-time in a spacelike direction. Consider two $\mathbb{Z}_2$ monopoles joined by a Dirac string made invisible through the use of appropriately twisted plaquettes. If their spacelike separation is large and they propagate in a spacelike direction, the $\mathbb{Z}_2$ flux between them will generate a portion of such a $\mathbb{Z}_2$ domain wall, that is bounded by the monopole world lines. If we relabel the direction of propagation as the time direction, so that the short euclidean t direction becomes a spatial direction, then we can re-express the above as telling us that, within this three-dimensional space in which one of the spatial directions is short (and equal to $1/T$) but the temperature is now zero, the potential energy of two $\mathbb{Z}_2$ monopole sources is linearly confining. (The asymmetric spatial geometry will by itself enforce confinement, if only logarithmic, at large r.) Moreover the corresponding string tension is calculable in perturbation theory. At low T by contrast, we expect any such flux tubes to condense in the vacuum because the system is confining, so that the monopoles experience a screened Coulomb interaction just as in $2+1$ dimensions.

The aspect of the present calculations that could be readily improved is that of the screening masses. This would require calculations at smaller a so as to increase the resolution of the region in which the Coulomb/Yukawa potential is visible. If one were to consider not just the total action, but the action over various distances around each or both of the monopoles, the statistical accuracy of the calculations would be greatly increased as well.

Note added. As we were preparing to submit this paper for publication, a revised version of [1] appeared in which a linear fit to the high-T inter-monopole interaction is used. This was shown to be necessary in the recent, more accurate, $D = 3 + 1$ calculations of [6], and, as we have shown in this paper, the same is true in $D = 2+1$.

Acknowledgments

We are grateful to Chris Korthals Altes and Alex Kovner for stimulating discussions on issues related to those dealt with in this paper. This collaboration began when one of the authors (ZS) was visiting Oxford and he is grateful to the Royal Society for funding the visit, under their European Science Exchange Programme. His work has also been supported in part by the Hungarian National Research Fund OTKA T032501 and the Bolyai János Research Grant. AH and BL acknowledge their postdoctoral funding from PPARC.

References

[1] C. Hoelbling, C. Rebbi and V.A. Rubakov, *Free energy of an* SU(2) *monopole-antimonopole pair*, `hep-lat/0003010`.

[2] A. Hart, B. Lucini, Z. Schram and M. Teper, *Vortices and confinement in hot and cold* $D = 2 + 1$ *gauge theories*, *J. High Energy Phys.* **06** (2000) 040 [`hep-lat/0005010`].

[3] L.D. Debbio, A.D. Giacomo and B. Lucini, *Vortices, monopoles and confinement*, `hep-lat/0006028`.

[4] G. 't Hooft, *On the phase transition towards permanent quark confinement*, *Nucl. Phys.* **B 138** (1978) 1; *A property of electric and magnetic flux in nonabelian gauge theories*, *Nucl. Phys.* **B 153** (1979) 141.

[5] C.K. Altes, A. Michels, M. Stephanov and M. Teper, *Domain walls and perturbation theory in high temperature gauge theory:* SU(2) *in 2+1 dimensions*, *Phys. Rev.* **D 55** (1997) 1047 [`hep-lat/9606021`].

[6] P. de Forcrand, M. D'Elia and M. Pepe, *A study of the't Hooft loop in* SU(2) *Yang-Mills theory*, `hep-lat/0007034`.

[7] M. Srednicki and L. Susskind, SU(5) *monopoles on the lattice*, *Nucl. Phys.* **B 179** (1981) 239.

[8] M. Teper, SU(N) *gauge theories in* (2+1)*-dimensions*, *Phys. Rev.* **D 59** (1999) 014512 [hep-lat/9804008].

[9] M. Teper, *The finite temperature phase transition of* SU(2) *gauge fields in* $(2+1)$*-dimensions*, *Phys. Lett.* **B 313** (1993) 417.

[10] A. Hart and M. Teper, *Monopole clusters in abelian projected gauge theories*, *Phys. Rev.* **D 58** (1998) 014504 [hep-lat/9712003].